ELECTROMAGNETIC ENGINEERING EE325

INDEX

COULOMB [C]

A unit of electrical charge equal to one amp second, the charge on 6.21×10^{18} electrons, or one joule per volt.

COMPLEX NOTATION

$$ae^{jb} = (a\angle b)$$

where b may be in radians or degrees (if noted).

COMPLEX CONJUGATES

The complex conjugate of a number is simply that number with the sign changed on the imaginary part. This applies to both **rectangular and polar notation**. When conjugates are multiplied, the result is a scalar.

$$(a + jb)(a - jb) = a^2 + b^2$$

$$(A\angle B^\circ)(A\angle - B^\circ) = A^2$$

Other **properties of conjugates**:

$$(ABC + DE + F)^* = (A^* B^* C^* + D^* E^* + F^*)$$

$$(e^{-jB})^* = e^{+jB}$$

TRANSMISSION LINES

G*L* REFLECTION COEFFICIENT [V/V]

The **reflection coefficient** is a value from –1 to +1 which, when multiplied by the wave voltage, determines the amount of voltage reflected at one end of the transmission line.

$$\boxed{\Gamma_L = \rho e^{j\psi} = \frac{Z_L - Z_0}{Z_L + Z_0}} \text{ and } Z_L = Z_0 \frac{1+\Gamma_L}{1-\Gamma_L}$$

where: Z_L is the **load impedance**

Γ_L is the **load reflection coefficient**

ρ is the **reflection coefficient magnitude**

ψ is the **reflection coefficient phase**

$Z_0 = \sqrt{\frac{L}{C}}$ is the **characteristic impedance**

THE COMPLEX WAVE EQUATION

The **complex wave equation** is applicable when the excitation is sinusoidal and the circuit is under steady state conditions.

$$\boxed{\frac{d^2V(z)}{dz^2} = -\beta^2 V(z)}$$

where $\beta = \omega\sqrt{LC} = \frac{2\pi}{\lambda}$ is the **phase constant**

The *complex wave equation* above is a second-order ordinary differential equation commonly found in the analysis of physical systems. The general solution is:

$$\boxed{V(z) = V^+ e^{-j\beta z} + V^- e^{+j\beta z}}$$

where $e^{-j\beta z}$ and $e^{+j\beta z}$ represent wave propagation in the $+z$ and $-z$ directions respectively.

The same equation applies to current:

$$\boxed{I(z) = I^+ e^{-j\beta z} + I^- e^{+j\beta z}}$$

and

$$\boxed{I(z) = \frac{V^+ e^{-j\beta z} + V^- e^{+j\beta z}}{Z_0}}$$

where $Z_0 = \sqrt{L/C}$ is the **characteristic impedance** of the line. These equations represent the voltage and current **phasors**.

SHORT-CIRCUIT IMPEDANCE [Ω]

$$\boxed{Z_{sc} = jZ_0 \tan(\beta l)}$$

where: Z_0 is the **characteristic impedance**

$\beta = \omega\sqrt{LC} = \frac{2\pi}{\lambda}$ is the **phase constant**

l is the length of the line [m]

CONSTANTS

Avogadro's number [molecules/mole]	$N_A = 6.02 \times 10^{23}$
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ J/K
	$= 8.62 \times 10^{-5}$ eV/K
Elementary charge	$q = 1.60 \times 10^{-19}$ C
Electron mass	$m_0 = 9.11 \times 10^{-31}$ kg
Permittivity of free space	$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m
Permeability constant	$\mu_0 = 4\pi \times 10^{-7}$ H/m
Planck's constant	$h = 6.63 \times 10^{-34}$ J-s
	$= 4.14 \times 10^{-15}$ cV-s
Rydberg constant	$R = 109{,}678$ cm^{-1}
kT @ room temperature	$kT = 0.0259$ eV
Speed of light	$c = 2.998 \times 10^{8}$ m/s
1 Å (angstrom)	10^{-8} cm = 10^{-10} m
1 μm (micron)	10^{-4} cm

1 nm = 10Å = 10^{-7} cm

1 eV = 1.6 × 10^{-19} J

1 V = 1 J/C 1 N/C = 1 V/m 1 J = 1 N· m = 1 C· V

l WAVELENGTH [m]

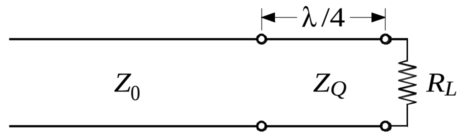
$$\boxed{\lambda = \frac{v_p}{f}}$$

v_p = velocity of propagation (2.998×10^8 m/s for a line in air)

f = frequency [Hz]

¼-WAVELENGTH INLINE MATCHING TRANSFORMER – resistive load

For use with a purely resistive load that does not match the line impedance. The load is matched to the line by inserting a ¼-wavelength segment having a characteristic impedance Z_Q.

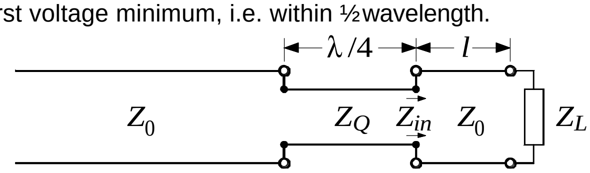


$$Z_Q = \sqrt{Z_0 R_L}$$

Z_0 = characteristic impedance of the transmission line [Ω]
λ = wavelength [meters]
R_L = resistance of the load [Ω]
Z_Q = characteristic impedance of the ¼wave matching segment [Ω]

¼-WAVELENGTH INLINE MATCHING TRANSFORMER – reactive load

For use with a reactive load. The load is matched to the line by inserting a ¼-wavelength segment having a characteristic impedance Z_Q at a distance l from the load. l is the length of transmission line required to produce the first voltage maximum—closest to the load. If the load is inductive, the first voltage maximum will be closer than the first voltage minimum, i.e. within ½wavelength.



First find the reflection coefficient in order to determine the value of ψ. Then find the length l of the line that will convert the load to a pure resistance, i.e. produces the first voltage maximum. Find this resistance (Z_{in}) using the line impedance formula. Then determine the impedance Z_Q of the ¼-wavelength segment that will match the load to the line.

$$\Gamma_L = \rho e^{j\psi} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

i.e. $\Gamma_L = \rho\angle\psi$ (radians)

$$l = \frac{\psi}{2\beta} = \frac{\psi\lambda}{4\pi}$$

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$$

$$Z_Q = \sqrt{Z_0 Z_{in}}$$

Γ_L is the load reflection coefficient
ψ = phase of the reflection coefficient [radians]
ρ = magnitude of the reflection coefficient [Ω]
Z_0 = characteristic impedance [Ω]
$\beta = 2\pi/\lambda$
λ = v_p/f wavelength [m]
Z_{in} = impedance (resistive) of the load combined with the l segment [Ω]
Z_Q = line impedance of the ¼-wave matching segment [Ω]

X REACTANCE [Ω]

$$X_C = \frac{-j}{\omega C}$$

$$X_L = j\omega L$$

X_C = reactance [Ω]
X_L = reactance [Ω]
$j = \sqrt{-1}$
ω = frequency [radians]
C = capacitance [F]
L = inductance [H]

Z_{in} LINE IMPEDANCE [Ω]

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$$

l = distance from load [m]
$j = \sqrt{-1}$
β = phase constant
Z_0 = characteristic impedance [Ω]
Z_L = load impedance [Ω]

The **line impedance** of a ¼-wavelength line is the inverse of the load impedance.

Impedance is a real value when its magnitude is maximum or minimum.

$$Z_{max} = Z_0 S = Z_0 \frac{1+\rho}{1-\rho}$$

$$Z_{min} = \frac{Z_0}{S} = Z_0 \frac{1-\rho}{1+\rho}$$

Z_0 = characteristic impedance [Ω]
S = standing wave ratio
ρ = magnitude of the reflection coefficient

SMITH CHARTS

First **normalize the load** impedance by dividing by the characteristic impedance, and find this point on the chart. An **inductive load** will be located on the top half of the chart, a **capacitive load** on the bottom half.

Draw a **straight line** from the center of the chart through the normalized load impedance point to the edge of the chart.

Anchor a compass at the center of the chart and **draw an arc** through the normalized load impedance point. Points along this arc represent the normalized impedance at various points along the transmission line. **Clockwise movement** along the arc represents movement from the load toward the source with one full revolution representing **1/2 wavelength** as marked on the outer circle. The two points where the arc intersects the horizontal axis are the **voltage maxima** (right) and the **voltage minima** (left).

Points opposite the impedance (180° around the arc) are **admittance**. The reason admittance is useful is because admittances in parallel are simply added.

$$\boxed{\Gamma(z)=\Gamma_L e^{j2\beta z}}$$

$$e^{j2\beta z}=1\angle 2\beta z$$

$$\Gamma(z)=\frac{\mathbf{Z}(z)-1}{\mathbf{Z}(z)+1}$$

$$\mathbf{Z}_L=\frac{\Gamma_L-1}{\Gamma_L+1} \qquad \boxed{\mathbf{Z}=\frac{Z_L}{Z_0}}$$

z = distance from load [m]

j = $\sqrt{-1}$

ρ = magnitude of the reflection coefficient

β = phase constant

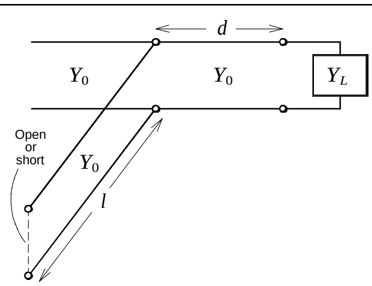
Γ = reflection coefficient

Z = normalized impedance [Ω]

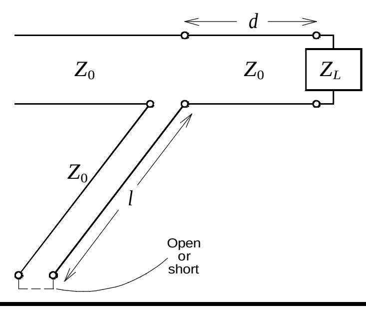
SINGLE-STUB TUNING

The basic idea is to connect a line stub in parallel (shunt) or series a distance *d* from the load so that the imaginary part of the load impedance will be canceled.

Shunt-stub: **Select *d* so that the admittance *Y* looking toward the load from a distance *d* is of the form $Y_0 + jB$. Then the stub susceptance is chosen as $-jB$, resulting in a matched condition.**

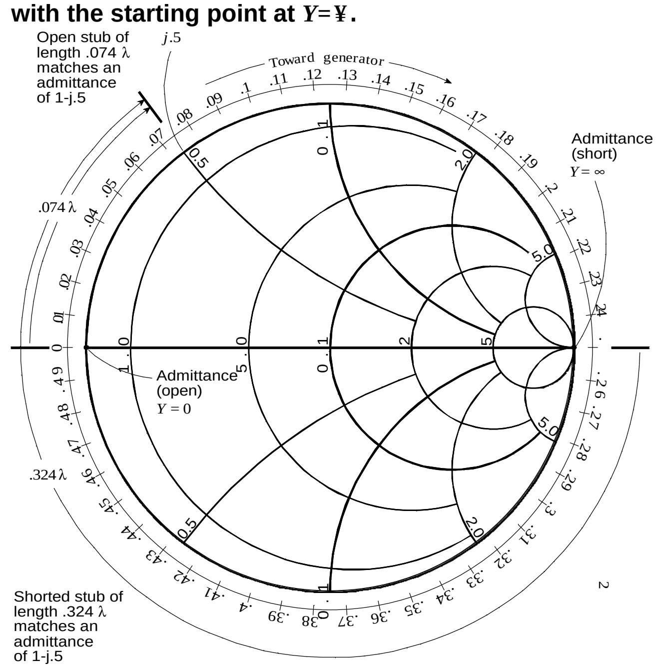


Series-stub: Select d so that the admittance Z looking toward the load from a distance d is of the form $Z_0 + jX$. Then the stub susceptance is chosen as $-jX$, resulting in a matched condition.



FINDING A STUB LENGTH

Example: Find the lengths of open and shorted shunt stubs to match an admittance of 1-j0.5. The admittance of an open shunt (zero length) is *Y*=0; this point is located at the left end of the Smith Chart *x*-axis. We proceed clockwise around the Smith chart, i.e. away from the end of the stub, to the +j0.5 arc (the value needed to match –j0.5). The difference in the starting point and the end point on the wavelength scale is the length of the stub in wavelengths. The length of a shorted-type stub is found in the same manner but with the starting point at *Y*=¥.



In this example, all values were in units of admittance. If we were interested in finding a stub length for a series stub problem, the units would be in impedance. The problem would be worked in exactly the same way. Of course in impedance, an open shunt (zero length) would have the value *Z*=¥, representing a point at the right end of the *x*-axis.

SWR STANDING WAVE RATIO [V/V]

$$\boxed{\mathrm{SWR}=\frac{|V|_{\max}}{|V|_{\min}}=\frac{|I|_{\max}}{|I|_{\min}}=\frac{1+\rho}{1-\rho}}$$

P(*z*) TIME-AVERAGE POWER ON A LOSSLESS TRANSMISSION LINE [W]

Equal to the **power** delivered to the load. The power delivered to the load is maximized under matched conditions, i.e. ρ = 0, otherwise part of the power is reflected back to the source. To calculate power, it may be simpler to find the input impedance and use $P = I^2R$ or $P = IV$.

$$\boxed{\mathrm{P}(z) = \frac{\left|V^+\right|^2}{2Z_0}\left(1-\rho^2\right)}$$

$$\mathrm{P}(z) = \frac{1}{2}\mathscr{Re}\left\{V(z)\ \ [I(z)]^*\right\}$$

V^+ = the voltage of the forward-traveling wave [V]
Z_0 = characteristic impedance [Ω]
ρ = magnitude of the reflection coefficient
$\mathscr{Re}$ = "the real part"

POWER USING PHASOR NOTATION [W]

$$\boxed{\mathbf{S} = \frac{1}{2}\mathbf{VI}^*}$$

S = power [W]
V = volts [V]
*I** = complex conjugate of current [A]

V+ FORWARD-TRAVELING WAVE

$$\boxed{V^+ = \frac{Z_{in}V_0}{(Z_{in}+Z_S)e^{j\beta l}\left(1+\Gamma_L e^{-j2\beta l}\right)}}$$

V^+ = the voltage of the forward-traveling wave [V]
V_0 = source voltage [V]
Z_{in} = input impedance [Ω]
Z_S = source impedance [Ω]
β = phase constant
l = length of the line [m]
Γ_L = load reflection coefficient

ELECTROSTATICS

F ELECTROSTATIC FORCE

$$\boxed{\mathbf{F}_{12} = \frac{1}{4\pi\varepsilon_0}Q_1Q_2\frac{(\mathbf{r}_2-\mathbf{r}_1)}{\left|\mathbf{r}_2-\mathbf{r}_1\right|^3}} \qquad \frac{1}{4\pi\varepsilon_0} = 9\times10^9$$

$\mathbf{F}_{12}$ = the force exerted **by** charge Q_1 **on** Q_2. [N]
$\mathbf{r}_1$ = vector from the origin to Q_1
$\mathbf{r}_2$ = vector from the origin to Q_2.

When finding the force on one charge due to multiple charges, the result can be found by summing the effects of each charge separately or by converting the multiple charges to a single equivalent charge and solving as a 2-charge problem.

E ELECTRIC FIELD

$$\mathbf{E}_p = \frac{1}{4\pi\varepsilon_0}\sum_{k=1}^{n}Q_k\frac{\mathbf{r}-\mathbf{r}_k'}{\left|\mathbf{r}-\mathbf{r}_k'\right|^3}$$

$$d\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\hat{\mathbf{R}}\frac{\rho_l(r')}{\left|\mathbf{r}-\mathbf{r}'\right|^2}dl'$$

$$\boxed{\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\int\hat{\mathbf{R}}\frac{\rho_l(r')}{\left|\mathbf{r}-\mathbf{r}'\right|^2}dl'}$$

Electric field from a potential:

$$\boxed{\mathbf{E} = -\nabla\Phi}$$

refer to the NABLA notes on page 8.

*NOTE: The *l* symbols could be replaced by a symbol for area or volume. See *Working With …* on page 9.

$\mathbf{E}_p$ = electric field at point *p* due to a charge *Q* or charge density ρ [V/m]
$d\mathbf{E}$ = an increment of electric field [V/m]
Q = electric charge [C]
ε_0 = permittivity of free space 8.85 × 10^{-12} F/m
ρ_l = charge density; charge per unit length* [C/m]
dl′ = a small segment of line *l**
$\hat{\mathbf{R}}$ = unit vector pointing from **r′** to **r** , i.e. in the direction of **r** - **r′**.
r′ = vector location of the **source** charge in relation to the origin
r = vector location of the point at which the value of $\mathbf{E}_p$ is observed
∇ = Del, Grad, or Nabla operator

F ELECTROSTATIC POTENTIAL [V]

$$\Phi = \frac{1}{4\pi\varepsilon_0}\sum_{k=1}^{n}\frac{Q_k}{\left|\mathbf{r}-\mathbf{r}_k'\right|}$$

$$d\Phi = \frac{1}{4\pi\varepsilon_0}\frac{\rho_l dl'}{\left|\mathbf{r}-\mathbf{r}'\right|}$$

$$\boxed{\Phi = \frac{1}{4\pi\varepsilon_0}\int\frac{\rho_l}{\left|\mathbf{r}-\mathbf{r}'\right|}dl'}$$

Potential due to an electric field:

$$\boxed{\Phi_{ab} = -\int_a^b \mathbf{E}\cdot d\mathbf{l}}$$

To evaluate voltage at all points.

$$\boxed{\Phi(r) = -\int_\infty^r \mathbf{E}\cdot d\mathbf{l}}$$

*NOTE: The *l* symbols could be replaced by a symbol for area or volume. See *Working With …* on page 9.

Φ = the potential [V]
*d*Φ = an increment of potential [V]
Φ_{ab} = the potential difference between points *a* and *b* [V]
E = electric field
dl′ = a small segment of line *l**
*d***l** = the differential vector displacement along the path from *a* to *b*
ε_0 = permittivity of free space 8.85 × 10^{-12} F/m
Q = electric charge [C]
ρ_l = charge density along a line* [C/m]
$\mathbf{r}_k'$ = vector location of **source** charge Q_k
r′ = vector location of the **source** charge in relation to the origin
r = vector location of electrostatic potential Φ in relation to the origin

MAXWELL'S EQUATIONS

Maxwell's equations govern the principles of guiding and propagation of electromagnetic energy and provide the foundations of all electromagnetic phenomena and their applications.

$$\nabla \times \bar{\mathscr{E}} = -\frac{\partial \bar{\mathscr{B}}}{\partial t}$$ Faraday's Law

$$\nabla \cdot \bar{D} = \bar{\rho}$$ Gauss' Law

$$\nabla \times \bar{\mathscr{H}} = \bar{\mathscr{J}} + \frac{\partial \bar{\mathscr{D}}}{\partial t}$$ Ampere's Law*

$$\nabla \cdot \bar{B} = 0$$ no name law, where:

$\mathscr{E}$ = electric field [V/m]
$\mathscr{B}$ = magnetic field [T]
t = time [s]
$\mathscr{D}$ = electric flux density [C/m²]
ρ = volume charge density [C/m³]
$\mathscr{H}$ = magnetic field intensity [A/m]
$\mathscr{J}$ = current density [A/m²]

*Maxwell added the $\frac{\partial \mathscr{D}}{\partial t}$ term to Ampere's Law.

POISSON'S EQUATION

$$\nabla^2 \Phi = -\frac{\rho}{\varepsilon_0}$$

r$_s$ SURFACE CHARGE DENSITY [C/m²]

$$\rho_s = \varepsilon_0 E_n$$

$$E_n = \hat{\mathbf{n}} \cdot \mathbf{E}$$

ε_0 = permittivity of free space 8.85×10^{-12} F/m
E_n = electric field normal to the surface [V/m]

D FLUX DENSITY [C/m²]

or ELECTRIC DISPLACEMENT PER UNIT AREA

$$\mathbf{D} \equiv \hat{\mathbf{r}} \frac{Q}{4\pi r^2}$$

$$\mathbf{D} = \varepsilon \mathbf{E}$$

Q = electric charge [C]
ε = dielectric constant $\varepsilon = \varepsilon_0 \varepsilon_r$
E = electric field [V/m]

GAUSS'S LAW

The net flux passing through a surface enclosing a charge is equal to the charge. Careful, what this first integral really means is the surface area multiplied by the perpendicular electric field. There may not be any integration involved.

$$\oint_S \varepsilon_0 \mathbf{E} \cdot d\mathbf{s} = Q_{enc} \qquad \oint_S \mathbf{D} \cdot d\mathbf{s} = \int_V \rho \, dv = Q_{enc}$$

ε_0 = permittivity of free space 8.85×10^{-12} F/m
E = electric field [V/m]
D = electric flux density vector [C/m²]
$d\mathbf{s}$ = a small increment of surface S
ρ = volume charge density [C/m³]
dv = a small increment of volume V
Q_{enc} = total electric charge enclosed by the Gaussian surface [S]

The differential version of Gauss's law is:

$$\nabla \cdot \mathbf{D} = \rho \quad \text{or} \quad \operatorname{div}(\varepsilon_0 \cdot \mathbf{E}) = \rho$$

GAUSS'S LAW – an example problem

Find the intensity of the electric field at distance r from a straight conductor having a voltage V.

Consider a cylindrical surface of length l and radius r enclosing a portion of the conductor. The electric field passes through the curved surface of the cylinder but not the ends. Gauss's law says that the electric flux passing through this curved surface is equal to the charge enclosed.

$$\oint_S \varepsilon_0 \mathbf{E} \cdot d\mathbf{s} = \varepsilon_0 \int_0^{2\pi} E_r lr \, d\phi = Q_{enc} = \rho_l l = C_l V l$$

so $\varepsilon_0 E_r r \int_0^{2\pi} d\phi = C_l V$ and $E_r = \frac{C_l V}{2\pi \varepsilon_0 r}$

E_r = electric field at distance r from the conductor [V/m]
l = length [m]
$r\,d$f = a small increment of the cylindrical surface S [m²]
ρ_l = charge density per unit length [C/m]
C_l = capacitance per unit length [F/m]
V = voltage on the line [V]

CONSERVATIVE FIELD LAW

$$\nabla \times \mathbf{E} = 0 \qquad \oint_S \mathbf{E} \cdot d\mathbf{l} = 0$$

E = electric field [V/m]
$d\mathbf{s}$ = a small increment of length

COULOMB'S LAW

$$\nabla \cdot \mathbf{D} = \rho \qquad \oint_S \mathbf{D} \cdot d\mathbf{s} = \int_V \rho \, dv$$

$\mathbf{D}$ = electric flux density vector [C/m^2]
ρ = volume charge density [C/m^3]
$d\mathbf{s}$ = a small increment of surface S

W_e POTENTIAL ENERGY [J]

The energy required to bring charge q from infinity to a distance R from charge Q.

$$W_e = q\Phi = \frac{Qq}{4\pi\varepsilon R}$$

$$W_e = \frac{1}{2}\int_V \rho\Phi \, dv = \frac{1}{2}\int_V \mathbf{D}\cdot\mathbf{E} \, dv$$

Φ = the potential between q and Q [V]
q,Q = electric charges [C]
ε = permittivity of the material
R = distance [m]
ρ = volume charge density [C/m^3]
$\mathbf{E}$ = electric field [V/m]
$\mathbf{D}$ = electric flux density vector [C/m^2]

w_e VOLUME ENERGY DENSITY [J/m^3] for the Electrostatic Field

$$w_e = \frac{1}{2}\mathbf{D}\cdot\mathbf{E} = \frac{1}{2}\varepsilon E^2$$

Φ = the potential between q and Q [V]
ε = permittivity of the material
R = distance [m]
$\mathbf{E}$ = electric field [V/m]
$\mathbf{D}$ = electric flux density vector [C/m^2]

CAPACITANCE

C CAPACITANCE [F]

$$C = \frac{Q}{\Phi}$$

$$C_l = \frac{\rho_l}{V}$$

Q = total electric charge [C]
Φ = the potential between q and Q [V]
C_l = capacitance per unit length [F/m]
ρ_l = charge density per unit length [C/m]
V = voltage on the line [V]

C CAPACITANCE BETWEEN TWO PARALLEL SOLID CYLINDRICAL CONDUCTORS

This also applies to a single conductor above ground, where the height above ground is $d/2$.

$$C = \frac{\pi\varepsilon}{\ln(d/a)}, \text{ where } d \gg a$$

or

$$C = \frac{\pi\varepsilon}{\cosh^{-1}\frac{d}{2a}}$$

C = capacitance [F/m]
ε = permittivity of the material
d = separation (center-to-center) [m]
a = conductor radius [m]

C CAPACITANCE BETWEEN PARALLEL PLATES

$$C = \frac{\varepsilon A}{d}$$

C = capacitance [F]
ε = permittivity of the material
d = separation of the plates [m]
A = area of one plate [m^2]

C CAPACITANCE BETWEEN COAXIAL CYLINDERS

$$C = \frac{2\pi\varepsilon}{\ln(b/a)}$$

C = capacitance [F/m]
ε = permittivity of the material
b = radius of the outer cylinder [m]
a = radius of the inner cylinder [m]

C CAPACITANCE OF CONCENTRIC SPHERES

$$C = \frac{4\pi\varepsilon ab}{b-a}$$

C = capacitance [F/m]
ε = permittivity of the material
b = radius of the outer sphere [m]
a = radius of the inner sphere [m]

J CURRENT DENSITY

The amount of current flowing perpendicularly through a unit area [A/m^2]

$$\mathbf{J} = \sigma\mathbf{E}$$

$$I = \oint_S \mathbf{J}\cdot d\mathbf{s}$$

In semiconductor material:

$$\mathbf{J} = n_c q_e \mathbf{v}_d$$

σ = conductivity of the material [S/m]
$\mathbf{E}$ = electric field [V/m]
I = current [A]
$d\mathbf{s}$ = a small increment of surface S
n_c = the number of conduction band electrons
q_e = electron charge -1.602×10^{-19} C
$\mathbf{v}_d$ = a small increment of surface S

CONTINUITY EQUATION

$$\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0$$

$\mathbf{J}$ = current density [A/m^2] $\mathbf{J} = \sigma \mathbf{E}$
ρ = volume charge density [C/m^3]

DUALITY RELATIONSHIP of J and D

RESISTANCE, CAPACITANCE, CURRENT, CONDUCTIVITY

Where current enters and leaves a conducting medium via two perfect conductors (electrodes) we have:

$$I = \oint_S \mathbf{J} \cdot d\mathbf{s} = \sigma \oint_S \mathbf{E} \cdot d\mathbf{s} = \frac{\sigma}{\varepsilon} \oint_S \mathbf{D} \cdot d\mathbf{s} = \frac{\sigma Q}{\varepsilon}$$

$\mathbf{J}$ = current density [A/m^2] $\mathbf{J} = \sigma \mathbf{E}$
$\mathbf{E}$ = electric field [V/m]
$\mathbf{D}$ = electric flux density vector [C/m^2] $\mathbf{D} = \varepsilon \mathbf{E}$

As a result of this, we have the following relation, useful in finding the resistance between two conductors:

$$RC = \frac{\varepsilon}{\sigma}$$

R = resistance [Ω]
C = capacitance [F]
ε = permittivity of the material
σ = conductivity of the material [S/m]

G CONDUCTANCE [Ω^{-1}]

$$G = \frac{1}{R} = \frac{I}{\Delta\Phi} = \frac{\sigma \oint_S \mathbf{E} \cdot d\mathbf{s}}{\int_+^- \mathbf{E} \cdot d\mathbf{l}}$$

R = resistance [Ω]
I = current [A]
ΔΦ = voltage potential [V]
σ = conductivity of the material [S/m]

s SEMICONDUCTOR CONDUCTIVITY [Ω^{-1}]

$$\sigma \approx |q| \mu_e N_d$$

σ = conductivity of the material [S/m]G = conductance [Ω^{-1}]
q = electron charge -1.602×10^{-19} C
μ_e = electron mobility [m^2/(V-s)]
N_d = concentration of donors, and thereby the electron concentration in the transition region [m^{-3}]

MATHEMATICS

WORKING WITH LINES, SURFACES, AND VOLUMES

$\rho_l(\mathbf{r}')$ means "the charge density along line l as a function of $\mathbf{r}'$." This might be a value in C/m or it could be a function. Similarly, $\rho_s(\mathbf{r}')$ would be the charge density of a surface and $\rho_v(\mathbf{r}')$ is the charge density of a volume.

For example, a disk of radius a having a uniform charge density of ρ C/m^2, would have a total charge of $\rho\pi a^2$, but to find its influence on points along the central axis we might consider incremental rings of the charged surface as $\rho_s(\mathbf{r}')\, dr' = \rho_s 2\pi r'\, dr'$.

If $d\mathbf{l}'$ refers to an incremental distance along a circular contour C, the expression is $r'd$f, where r' is the radius and df is the incremental angle.

GEOMETRY

SPHERE

Area $A = 4\pi r^2$

Volume $V = \frac{4}{3}\pi r^3$

ELLIPSE

Area $A = \pi AB$

Circumference

$$L \approx 2\pi \sqrt{\frac{a^2 + b^2}{2}}$$

Ñ NABLA, DEL OR GRAD OPERATOR [+ m^{-1}]

Compare the ∇ operation to taking the time derivative. Where $\partial/\partial t$ means to take the derivative with respect to time and introduces a s^{-1} component to the units of the result, the ∇ operation means to take the derivative with respect to distance (in 3 dimensions) and introduces a m^{-1} component to the units of the result. ∇ terms may be called ***space derivatives*** and an equation which contains the ∇ operator may be called a ***vector differential equation***. In other words $\nabla\mathbf{A}$ is how fast $\mathbf{A}$ changes as you move through space.

in rectangular coordinates:
$$\nabla \mathbf{A} = \hat{x}\frac{\partial A}{\partial x} + \hat{y}\frac{\partial A}{\partial y} + \hat{z}\frac{\partial A}{\partial z}$$

in cylindrical coordinates:
$$\nabla \mathbf{A} = \hat{r}\frac{\partial A}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial A}{\partial \phi} + \hat{z}\frac{\partial A}{\partial z}$$

in spherical coordinates:
$$\nabla \mathbf{A} = \hat{r}\frac{\partial A}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial A}{\partial \theta} + \hat{\phi}\frac{1}{r \sin\theta}\frac{\partial A}{\partial \phi}$$

Ñ^2 THE LAPLACIAN [+ m^{-2}]

in **rectangular** coordinates:

$$\nabla^2\mathbf{A} = \hat{\mathbf{x}}\nabla^2 A_x + \hat{\mathbf{y}}\nabla^2 A_y + \hat{\mathbf{z}}\nabla^2 A_z = 0$$

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = 0$$

in **spherical** and **cylindrical** coordinates:

$$\nabla^2\mathbf{A} \equiv \nabla(\nabla\cdot\mathbf{A}) - \nabla\times\nabla\times\mathbf{A}$$
$$= \text{grad}(\text{div }\mathbf{A}) - \text{curl}(\text{curl }\mathbf{A})$$

for example, the Laplacian of electrostatic potential:

$$\nabla^2\Phi = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2} = 0$$

Ñ· DIVERGENCE [+ m^{-1}]

The del operator followed by the dot product operator is read as "the divergence of" and is an operation performed on a vector. In rectangular coordinates, $\nabla\cdot$ means the sum of the partial derivatives of the magnitudes in the x, y, and z directions with respect to the x, y, and z variables. The result is a scalar, and a factor of m^{-1} is contributed to the units of the result.

For example, in this form of Gauss' law, where **D** is a density per unit area, $\nabla\cdot\mathbf{D}$ becomes a density per unit volume.

$$\boxed{\text{div }\mathbf{D} = \nabla\cdot\mathbf{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \rho}$$

D = electric flux density vector $\mathbf{D} = \varepsilon\mathbf{E}$ [C/m^2]
ρ = source charge density [C/m^3]

In the electrostatic context, the divergence of **D** is the total outward flux per unit volume due to a source charge. The divergence of vector **D** is:

in **rectangular** coordinates:

$$\text{div }\mathbf{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

in **cylindrical** coordinates:

$$\text{div }\mathbf{D} = \frac{1}{r}\frac{\partial}{\partial r}(rD_r) + \frac{1}{r}\frac{\partial D_\phi}{\partial\phi} + \frac{\partial D_z}{\partial z}$$

in **spherical** coordinates:

$$\text{div }\mathbf{D} = \frac{1}{r^2}\frac{\partial(r^2 D_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta D_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi}$$

Ñ× CURL [+ m^{-1}]

The circulation around an enclosed area. The curl of vector **B** is
in **rectangular** coordinates:

$$\text{curl }\mathbf{B} = \nabla\times\mathbf{B} =$$
$$\hat{x}\left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right) + \hat{y}\left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right) + \hat{z}\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right)$$

in **cylindrical** coordinates:

$$\text{curl }\mathbf{B} = \nabla\times\mathbf{B} =$$
$$\hat{r}\left[\frac{1}{r}\frac{\partial B_z}{\partial\phi} - \frac{\partial B_\phi}{\partial z}\right] + \hat{\phi}\left[\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r}\right] + \hat{z}\frac{1}{r}\left[\frac{\partial(rB_\phi)}{\partial r} - \frac{\partial B_r}{\partial\phi}\right]$$

in **spherical** coordinates:

$$\text{curl }\mathbf{B} = \nabla\times\mathbf{B} = \hat{r}\,\frac{1}{r\sin\theta}\left[\frac{\partial(B_\phi\sin\theta)}{\partial\theta} - \frac{\partial B_\theta}{\partial\phi}\right] +$$
$$\hat{\theta}\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial B_r}{\partial\phi} - \frac{\partial(rB_\phi)}{\partial r}\right] + \hat{\phi}\frac{1}{r}\left[\frac{\partial(rB_\theta)}{\partial r} - \frac{\partial B_r}{\partial\theta}\right]$$

The **divergence of a curl** is always zero:

$$\nabla\cdot(\nabla\times\mathbf{H}) = 0$$

DOT PRODUCT [= units^2]

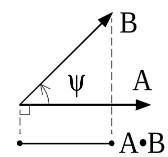
The dot product is a scalar value.

$$\mathbf{A}\bullet\mathbf{B} = (\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z)\bullet(\hat{\mathbf{x}}B_x + \hat{\mathbf{y}}B_y + \hat{\mathbf{z}}B_z) = A_xB_x + A_yB_y + A_zB_z$$

$$\mathbf{A}\bullet\mathbf{B} = |\mathbf{A}||\mathbf{B}|\cos\psi_{AB}$$

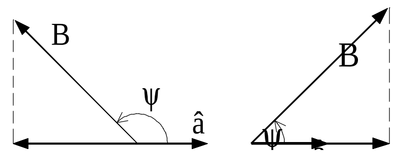
$$\hat{\mathbf{x}}\bullet\hat{\mathbf{y}} = 0,\quad \hat{\mathbf{x}}\bullet\hat{\mathbf{x}} = 1$$

$$\mathbf{B}\bullet\hat{\mathbf{y}} = (\hat{\mathbf{x}}B_x + \hat{\mathbf{y}}B_y + \hat{\mathbf{z}}B_z)\bullet\hat{\mathbf{y}} = B_y$$



Projection of **B** along **â**:

$$(\mathbf{B}\bullet\hat{\mathbf{a}})\hat{\mathbf{a}}$$



The dot product of 90° vectors is zero.
The dot product is **commutative** and **distributive**:

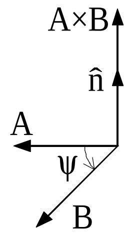
$$\mathbf{A}\bullet\mathbf{B} = \mathbf{B}\bullet\mathbf{A} \qquad \mathbf{A}\bullet(\mathbf{B}+\mathbf{C}) = \mathbf{A}\bullet\mathbf{B} + \mathbf{A}\bullet\mathbf{C}$$

CROSS PRODUCT

$$\mathbf{A}\times\mathbf{B} = \left(\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z\right)\times\left(\hat{\mathbf{x}}B_x + \hat{\mathbf{y}}B_y + \hat{\mathbf{z}}B_z\right)$$

$$= \hat{\mathbf{x}}\left(A_yB_z - A_zB_y\right) + \hat{\mathbf{y}}\left(A_zB_x - A_xB_z\right) + \hat{\mathbf{z}}\left(A_xB_y - A_yB_x\right)$$

$$\mathbf{A}\times\mathbf{B} = \hat{\mathbf{n}}\left|\mathbf{A}\right|\left|\mathbf{B}\right|\sin\psi_{AB}$$



where $\hat{\mathbf{n}}$ is the unit vector normal to both **A** and **B** (thumb of right-hand rule).

$$\mathbf{B}\times\mathbf{A} = -\mathbf{A}\times\mathbf{B}$$

$\mathbf{x}\times\mathbf{y} = \mathbf{z}$ $\quad$ $\mathbf{y}\times\mathbf{x} = -\mathbf{z}$ $\quad$ $\mathbf{x}\times\mathbf{x} = 0$

$\boldsymbol{\phi}\times\mathbf{z} = \mathbf{r}$ $\quad$ $\boldsymbol{\phi}\times\mathbf{r} = -\mathbf{z}$

The cross product is **distributive:**

$$\mathbf{A}\times\left(\mathbf{B}+\mathbf{C}\right) = \mathbf{A}\times\mathbf{B} + \mathbf{A}\times\mathbf{C}$$

Also, we have:

$$\mathbf{A}\times\left(\mathbf{B}\times\mathbf{C}\right) = \left(\mathbf{A}\cdot\mathbf{C}\right)\mathbf{B} - \left(\mathbf{A}\cdot\mathbf{B}\right)\mathbf{C}$$

COORDINATE SYSTEMS

Cartesian or Rectangular Coordinates:

$\mathbf{r}(x, y, z) = x\hat{x} + y\hat{y} + z\hat{z}$ $\qquad$ $\hat{x}$ is a unit vector

$$\left|\mathbf{r}\right| = \sqrt{x^2 + y^2 + z^2}$$

Spherical Coordinates:

$\mathbf{P}(r, \theta, \phi)$ $\quad$ r is distance from center

θ is angle from vertical

ϕ is the CCW angle from the x-axis

$\hat{r}$, $\hat{\theta}$, and $\hat{\phi}$ are unit vectores and are functions of position—their orientation depends on where they are located.

Cylindrical Coordinates:

$\mathbf{C}(r, \phi, z)$ $\quad$ r is distance from the vertical (z) axis

ϕ is the CCW angle from the x-axis

z is the vertical distance from origin

COORDINATE TRANSFORMATIONS

Rectangular to Cylindrical:

To obtain: $\mathbf{A}(r, \phi, z) = \hat{r}A_r + \hat{\phi}A_\phi + \hat{z}A_z$

$A_r = \sqrt{x^2 + y^2}$ $\qquad$ $\hat{r} = \hat{x}\cos\phi + \hat{y}\sin\phi$

$\phi = \tan^{-1}\dfrac{y}{x}$ $\qquad$ $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$

$z = z$ $\qquad$ $\hat{z} = \hat{z}$

Cylindrical to Rectangular:

To obtain: $\mathbf{r}(x, y, z) = x\hat{x} + y\hat{y} + z\hat{z}$

$x = r\cos\phi$ $\qquad$ $\hat{x} = \hat{r}\cos\phi - \hat{\phi}\cos\phi$

$y = r\sin\phi$ $\qquad$ $\hat{\phi} = \hat{r}\sin\phi + \hat{y}\cos\phi$

$z = z$ $\qquad$ $\hat{z} = \hat{z}$

Rectangular to Spherical:

To obtain: $\mathbf{A}(r, \theta, \phi) = \hat{r}A_r + \hat{\theta}A_\theta + \hat{\phi}A_\phi$

$A_r = \sqrt{x^2 + y^2 + z^2}$

$\hat{r} = \hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta$

$$\theta = \frac{z\cos^{-1}}{\sqrt{x^2 + y^2 + z^2}}$$

$\hat{\theta} = \hat{x}\cos\theta\cos\phi + \hat{y}\cos\theta\sin\phi - \hat{z}\sin\theta$

$\phi = \tan^{-1}\dfrac{y}{x}$ $\qquad$ $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$

Spherical to Rectangular:

To obtain: $\mathbf{r}(x, y, z) = x\hat{x} + y\hat{y} + z\hat{z}$

$x = r\sin\theta\cos\phi$

$\hat{x} = \hat{r}\sin\theta\cos\phi - \hat{\theta}\cos\theta\cos\phi - \hat{\phi}\sin\phi$

$y = r\sin\theta\sin\phi$

$\hat{y} = \hat{r}\sin\theta\sin\phi + \hat{\theta}\cos\theta\sin\phi + \hat{\phi}\cos\phi$

$z = r\cos\theta$ $\qquad$ $\hat{z} = \hat{r}\cos\theta - \hat{\theta}\sin\theta$

THE STATIC MAGNETIC FIELD

F $\mathbf{F}_{12}$ MAGNETIC FORCE [N/m] due to a conductor

If the current in the two wires travels in opposite directions, the force will be attractive.

$$\mathbf{F}_{12} = \frac{\hat{\mathbf{x}}\mu_0 I_1 I_2}{2\pi d}$$

$\mathbf{F}_{12}$ = the force exerted **by** conductor 1 carrying current I **on** conductor 2. [N/m]
μ_0 = permeability constant $4\pi\times10^{-7}$ [H/m]
I = current [A]
d = distance between conductors [m]

$\mathbf{B}_P$ BIOT-SAVART LAW

Determines the **B** field vector at any point P identified by the position vector **r**, due to a differential current element $I\,d\mathbf{l}'$ located at vector **r'**.

$$d\mathbf{B}_P = \frac{\mu_0 I\ d\mathbf{l}'\times\hat{\mathbf{R}}}{4\pi R^2}$$

$$\mathbf{B}_P = \frac{\mu_0 I}{4\pi}\oint_C \frac{d\mathbf{l}'\times(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}$$

$$\hat{\mathbf{R}} = \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}$$

$\mathbf{B}_P$ = magnetic field vector [T]
μ_0 = permeability constant $4\pi\times10^{-7}$ [H/m]
$I\,d\mathbf{l}'$ = current element [A]
$\hat{\mathbf{R}}$ = unit vector pointing from the current element to point P
R = distance between the current element and point P [m]

B AMPERE'S CIRCUITAL LAW

Ampere's law is a consequence of the **Biot-Savart law** and serves the same purpose as **Gauss's law**. Ampere's law states that the line integral of **B** around any closed contour is equal to μ_0 times the total net current I passing through the surface S enclosed by the contour C. This law is useful in solving magnetostatic problems having some degree of symmetry.

$$\oint_C \mathbf{B}\cdot d\mathbf{l} = \int_S \mu_0 \mathbf{J}\cdot d\mathbf{s} = \mu_0 I$$

B = magnetic field vector, equal to B times the appropriate unit vector [T]
μ_0 = permeability constant $4\pi\times10^{-7}$ [H/m]
$d\mathbf{l}$ = an increment of the line which is the perimeter of contour C [m]
J = current density [A/m^2] $\mathbf{J} = \sigma\mathbf{E}$
$d\mathbf{s}$ = an increment of surface [m^2]

B MAGNETIC FIELD [T or A/m] due to an infinite straight conductor

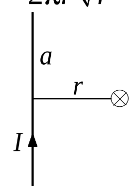
May also be applied to the magnetic field close to a conductor of finite length.

$$\mathbf{B}_P = \hat{\phi}\frac{\mu_0 I}{2\pi r}$$

$\mathbf{B}_P$ = magnetic field vector [T]
μ_0 = permeability constant $4\pi\times10^{-7}$ [H/m]
I = current [A]
r = perpendicular distance from the conductor [m]

B MAGNETIC FIELD [T] due to a finite straight conductor at a point perpendicular to the midpoint

$$\mathbf{B}_P = \hat{\phi}\frac{\mu_0 I a}{2\pi r\sqrt{r^2+a^2}}$$



$\mathbf{B}_P$ = magnetic field vector [T]
μ_0 = permeability constant $4\pi\times10^{-7}$ [H/m]
I = current [A]
a = half the length of the conductor [m]
r = perpendicular distance from the conductor [m]

B MAGNETIC FIELD [T] at the center of a circular wire of N turns

$$B_{ctr} = \hat{\mathbf{z}}\frac{\mu_0 NI}{2a}$$

B = magnetic field [T]
μ_0 = permeability const. $4\pi\times10^{-7}$ [H/m]
N = number of turns of the coil
I = current [A]
a = radius [m]

B MAGNETIC FIELD [T] along the central axis of a solenoid

$$B(z) = \hat{\mathbf{z}}\frac{\mu_0 NI}{2l}\left[\frac{(z+l/2)}{\sqrt{a^2+(z+l/2)^2}} - \frac{(z-l/2)}{\sqrt{a^2+(z-l/2)^2}}\right]$$

and at the center of the coil: $B_{ctr} \approx \hat{\mathbf{z}}\frac{\mu_0 NI}{l}$

B = magnetic field [T]
μ_0 = permeability constant $4\pi\times10^{-7}$ [H/m]
N = number of turns
I = current [A]
l = length of the solenoid [m]
z = distance from center of the coil [m]
a = coil radius [m]

H MAGNETIC FIELD INTENSITY [A/m]

The **magnetic field intensity vector** is directly analogous to the *electric flux density vector* **D** in electrostatics in that both **D** and **H** are medium-independent and are directly related to their sources.

$$\mathbf{H} \equiv \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

H = magnetic field [A/m]
B = magnetic field vector [T]
μ_0 = permeability const. $4\pi\times10^{-7}$ [H/m]
M = magnetization [A/m]
J = current density [A/m^2] $\mathbf{J} = \sigma\mathbf{E}$
D = electric flux density vector [C/m^2]

Y, L (,lambda) MAGNETIC FLUX, LINKAGE

Flux linkage Λ is the ability of a closed circuit to **store magnetic energy**. It depends, in part, on the physical layout of the conductors. It is the total magnetic field due to circuit #1 passing through the area enclosed by the conductors of circuit #2. The text seemed to describe Ψ as the flux due to one turn and Λ as the flux due to all of the turns of the coil, but was not consistent so be careful.

$$\boxed{\Psi_{12} = \int_{S_2} \mathbf{B}_1 \cdot d\mathbf{s}_2}$$

$$\Lambda_{12} = N_1 \Psi_{12}$$

$$\Lambda = N \int_S \mathbf{B} \cdot d\mathbf{s}$$

Ψ_{12} = the magnetic flux passing through coil 2 that is produced by a current in coil 1 [Wb]
Λ = total flux linkage [Wb]
B = magnetic field vector [T]
N = number of turns of the coil
$d\mathbf{s}$ = an increment of surface [m^2]

LENZ'S LAW

Induced voltage causes current to flow in the direction that produces a magnetic flux which **opposes** the flux that induced the voltage in the first place. This law is useful in checking or determining the sign or polarity of a result.

L INDUCTANCE [H]

Inductance is the ability of a conductor configuration to "link magnetic flux", i.e. store magnetic energy. Two methods of calculating inductance are given below.

$$L = \frac{\Lambda}{I}$$

$$L = \frac{2W_m}{I^2}$$

Λ = flux linkage [Wb]
I = current [A]
W_m = energy stored in a magnetic field [J]

L_{11} SELF-INDUCTANCE [H]

When a current in coil 1 induces a current in coil 2, the induced current in coil 2 induces a current back in coil 1. This is self-inductance.

$$L_{11} = \frac{N_1 \Lambda_{11}}{I_1} = \frac{N_1^{\,2} \Psi_{11}}{I_1}$$

N = number of turns of the coil
Λ_{11} = the total flux linked by a single turn of coil 1 [Wb]
I_1 = current in coil 1 [A]
Ψ_{11} = the magnetic flux produced by a single turn of coil 1 and linked by a single turn of coil 1 [Wb]

L_{12} MUTUAL INDUCTANCE [H]

The mutual inductance between two coils.

$$L_{12} = \frac{N_2 \Lambda_{12}}{I_1} = \frac{N_2 N_1 \Psi_{12}}{I_1}$$

Neumann formula:

$$L_{12} = \frac{\mu_0 N_1 N_2}{4\pi} \oint_{C_1} \oint_{C_2} \frac{d\mathbf{l}_1 \cdot d\mathbf{l}_2}{\left|\mathbf{r} - \mathbf{r}'\right|}$$

N = number of turns of the coil
Λ = flux linkage [Wb]
I = current [A]
Ψ = magnetic flux [Wb]
r = vector to the point of observation
r' = vector to source

W_m MAGNETIC ENERGY [J]

Energy stored in a magnetic field [Joules].

$$W_m = \frac{1}{2\mu_0} \int_V B^2 dv'$$

W_m = energy stored in a magnetic field [J]
μ_0 = permeability constant $4\pi\times10^{-7}$ [H/m]
B = magnetic field [T]

FARADAY'S LAW

When the magnetic flux enclosed by a loop of wire changes with time, a current is produced in the loop. The variation of the magnetic flux can result from a time-varying magnetic field, a coil in motion, or both.

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$\nabla\times\mathbf{E}$ = the curl of the electric field
B = magnetic field vector [T]

Another way of expressing Faraday's law is that a changing magnetic field induces an electric field.

$$V_{ind} = \oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{s}$$

where S is the surface enclosed by contour C.

(see also Induced Voltage below)

V_{ind} INDUCED VOLTAGE

The voltage induced in a coil due to a **changing magnetic field** is equal to the number of turns in the coil times the rate at which the magnetic field is changing (could be a change in field strength or coil area normal to the field).

$$V_{ind} = -N\frac{d\Psi}{dt}$$

$$V_{ind} = \oint_C \mathbf{E}\cdot d\mathbf{l}$$

N = number of turns of the coil

Ψ = the magnetic flux produced by a single turn of the coil [Wb]

V_{ind} INDUCED VOLTAGE DUE TO MOTION

When conductors move in the presence of magnetic fields, an induced voltage is produced even if the magnetic fields do not vary in time. For the voltage produced due to both a changing magnetic field and a conductor in motion:

$$V_{ind} = -\int_S \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{s} + \oint_C (\mathbf{v}\times\mathbf{B})\cdot d\mathbf{l}$$

$\mathbf{B}$ = magnetic field vector [T]

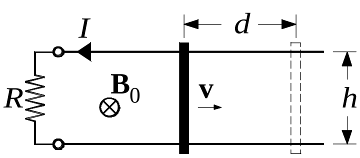
$\mathbf{v}$ = velocity vector of the conductor [m/s]

$d\mathbf{s}$ = increment of the surface normal to the magnetic field vector [m^2]

$d\mathbf{l}$ = incremental length of conductor [m]

INDUCED VOLTAGE – SLIDER PROBLEM

A frictionless conducting bar moves to the right at velocity v produces a current I.



An expanding magnetic field area having a static magnetic field directed into the page produces a CCW current.

$$\boxed{V_{ind} = B_0 h v}$$

$$\mathbf{F}_{mag} = \hat{\mathbf{x}} B_0 I h$$

$$E = I^2 R \frac{d}{v}$$

V_{ind} = induced voltage [V]

B_0 = static magnetic field [T]

h = distance between the conductor rails [T]

v = velocity of the conductor [m/s]

$\mathbf{F}_{mag}$ = magnetic force opposing slider [N]

$\hat{\mathbf{x}}$ = unit vector in the direction against conductor movement [m/s]

I = current [A]

E = energy produced [J or W/s]

R = circuit resistance [Ω]

d = distance the conductor moves [m]

M MAGNETIZATION [A/m]

The induced magnetic dipole moment per unit volume.

$$\mathbf{M} = -\frac{N q_e^2 a^2 \mathbf{B}}{4 m_e}$$

or $$\mathbf{M} = \frac{\chi_m \mathbf{B}}{\mu_0}$$

where

$$\chi_m = -\frac{N q_e^2 a^2 \mu_0}{4 m_e}$$

N = number of turns of the coil

q_e = electron charge - 1.602×10^{-19} C

a = orbit radius of an electron [m]

$\mathbf{B}$ = magnetic field vector [T]

μ_0 = permeability constant $4\pi\times10^{-7}$ [H/m]

m_e = who knows?

χ_m = magnetic susceptibility